

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MATHEMATICS

4725

Further Pure Mathematics 1

Thursday

8 JUNE 2006

Morning

1 hour 30 minutes

Additional materials:

8 page answer booklet

Graph paper

List of Formulae (MF1)

TIME 1 hour 30 minutes**INSTRUCTIONS TO CANDIDATES**

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- The total number of marks for this paper is 72.
- Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.
- **You are reminded of the need for clear presentation in your answers.**

This question paper consists of 3 printed pages and 1 blank page.

1 The matrices \mathbf{A} and \mathbf{B} are given by $\mathbf{A} = \begin{pmatrix} 4 & 1 \\ 0 & 2 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 1 & 1 \\ 0 & -1 \end{pmatrix}$.

(i) Find $\mathbf{A} + 3\mathbf{B}$. [2]

(ii) Show that $\mathbf{A} - \mathbf{B} = k\mathbf{I}$, where \mathbf{I} is the identity matrix and k is a constant whose value should be stated. [2]

2 The transformation S is a shear parallel to the x -axis in which the image of the point $(1, 1)$ is the point $(0, 1)$.

(i) Draw a diagram showing the image of the unit square under S . [2]

(ii) Write down the matrix that represents S . [2]

3 One root of the quadratic equation $x^2 + px + q = 0$, where p and q are real, is the complex number $2 - 3i$.

(i) Write down the other root. [1]

(ii) Find the values of p and q . [4]

4 Use the standard results for $\sum_{r=1}^n r^3$ and $\sum_{r=1}^n r^2$ to show that, for all positive integers n ,

$$\sum_{r=1}^n (r^3 + r^2) = \frac{1}{12}n(n+1)(n+2)(3n+1). \quad [5]$$

5 The complex numbers $3 - 2i$ and $2 + i$ are denoted by z and w respectively. Find, giving your answers in the form $x + iy$ and showing clearly how you obtain these answers,

(i) $2z - 3w$, [2]

(ii) $(iz)^2$, [3]

(iii) $\frac{z}{w}$. [3]

6 In an Argand diagram the loci C_1 and C_2 are given by

$$|z| = 2 \quad \text{and} \quad \arg z = \frac{1}{3}\pi$$

respectively.

(i) Sketch, on a single Argand diagram, the loci C_1 and C_2 . [5]

(ii) Hence find, in the form $x + iy$, the complex number representing the point of intersection of C_1 and C_2 . [2]

7 The matrix \mathbf{A} is given by $\mathbf{A} = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$.

(i) Find \mathbf{A}^2 and \mathbf{A}^3 . [3]

(ii) Hence suggest a suitable form for the matrix \mathbf{A}^n . [1]

(iii) Use induction to prove that your answer to part (ii) is correct. [4]

8 The matrix \mathbf{M} is given by $\mathbf{M} = \begin{pmatrix} a & 4 & 2 \\ 1 & a & 0 \\ 1 & 2 & 1 \end{pmatrix}$.

(i) Find, in terms of a , the determinant of \mathbf{M} . [3]

(ii) Hence find the values of a for which \mathbf{M} is singular. [3]

(iii) State, giving a brief reason in each case, whether the simultaneous equations

$$\begin{aligned} ax + 4y + 2z &= 3a, \\ x + ay &= 1, \\ x + 2y + z &= 3, \end{aligned}$$

have any solutions when

(a) $a = 3$,

(b) $a = 2$.

[4]

9 (i) Use the method of differences to show that

$$\sum_{r=1}^n \{(r+1)^3 - r^3\} = (n+1)^3 - 1. \quad [2]$$

(ii) Show that $(r+1)^3 - r^3 \equiv 3r^2 + 3r + 1$. [2]

(iii) Use the results in parts (i) and (ii) and the standard result for $\sum_{r=1}^n r$ to show that

$$3 \sum_{r=1}^n r^2 = \frac{1}{2}n(n+1)(2n+1). \quad [6]$$

10 The cubic equation $x^3 - 2x^2 + 3x + 4 = 0$ has roots α , β and γ .

(i) Write down the values of $\alpha + \beta + \gamma$, $\alpha\beta + \beta\gamma + \gamma\alpha$ and $\alpha\beta\gamma$. [3]

The cubic equation $x^3 + px^2 + 10x + q = 0$, where p and q are constants, has roots $\alpha + 1$, $\beta + 1$ and $\gamma + 1$.

(ii) Find the value of p . [3]

(iii) Find the value of q . [5]

